



Formula Booklet

Mathematics: Analysis & Approaches

Standard Level (SL)

For use during the course and in examinations
Based on IB Diploma Programme · First examinations 2021

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Topic 1: Number and Algebra

1.2 — Arithmetic Sequences & Series

Formula

$$u_n = u_1 + (n - 1)d$$

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d)$$

$$S_n = \frac{n}{2}(u_1 + u_n)$$

Explanation / Variables

n -th term. u_1 = first term, d = common difference.

Sum of n terms. Use when the last term is unknown.

Sum of n terms. Use when first and last term are known.

1.3 — Geometric Sequences & Series

Formula

$$u_n = u_1 \cdot r^{n-1}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r} \quad (r \neq 1)$$

$$S_\infty = \frac{u_1}{1 - r} \quad (|r| < 1)$$

Explanation / Variables

n -th term. r = common ratio.

Sum of n terms of a finite geometric sequence.

Sum of an **infinite** geometric sequence. Only valid when $|r| < 1$.

1.4 — Compound Interest

Formula

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

Explanation / Variables

FV = future value, PV = present value, $r\%$ = nominal annual rate, k = periods/year, n = years.

1.5 — Exponents & Logarithms (Basic)

Formula

$$\log_a x = b \Leftrightarrow a^b = x$$

Explanation / Variables

Definition of a logarithm. $a > 0$, $a \neq 1$, $x > 0$.

1.7 — Logarithm Laws

Formula

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^m) = m \log_a x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$a^x = e^{x \ln a}; \quad \log_a x = \frac{\ln x}{\ln a}$$

Explanation / Variables

Product rule: log of a product = sum of logs.

Quotient rule: log of a quotient = difference of logs.

Power rule: exponent moves in front as a factor.

Change of base formula. Useful when base is not 10 or e .

Conversion between exponential bases and natural log.

1.9 — Binomial Theorem**Formula**

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Explanation / Variables

Expansion of $(a + b)^n$ for $n \in \mathbb{N}$.

Binomial coefficient — number of ways to choose r from n .

Topic 2: Functions**2.1 — Straight Lines****Formula**

$$y = mx + c \quad ; \quad ax + by + d = 0 \quad ;$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Explanation / Variables

Three forms of a straight line. m = gradient, c = y -intercept.

Gradient between two points (x_1, y_1) and (x_2, y_2) .

2.6 — Quadratic Functions**Formula**

$$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$$

Explanation / Variables

Axis of symmetry / x -coordinate of vertex.

2.7 — Quadratic Equations**Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

Explanation / Variables

Quadratic formula. Solves $ax^2 + bx + c = 0$, $a \neq 0$.

Discriminant. $\Delta > 0$: two real roots; $\Delta = 0$: one repeated root; $\Delta < 0$: no real roots.

Topic 3: Geometry and Trigonometry

Prior Learning — Areas & Volumes

Formula

$$A = bh$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(a + b)h$$

$$A = \pi r^2; \quad C = 2\pi r$$

$$V = lwh; \quad V = \pi r^2 h; \quad V = Ah$$

$$A = 2\pi r h$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Explanation / Variables

Area of a parallelogram. b = base, h = perpendicular height.

Area of a triangle.

Area of a trapezoid. a, b = parallel sides.

Area and circumference of a circle.

Volumes of cuboid, cylinder, prism.

Curved surface area of a cylinder.

Distance between two points in 2D.

Midpoint in 2D.

3.1 — 3D Geometry

Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Explanation / Variables

Distance between two points in 3D.

Midpoint in 3D.

3D Solids

Formula

$$V = \frac{1}{3}Ah; \quad V = \frac{1}{3}\pi r^2 h$$

$$A = \pi r l$$

$$V = \frac{4}{3}\pi r^3; \quad A = 4\pi r^2$$

Explanation / Variables

Volume of pyramid and cone. A = base area.

Curved surface of cone. l = slant height.

Volume and surface area of sphere.

3.2 — Trigonometry

Formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \quad ;$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$A = \frac{1}{2}ab \sin C$$

Explanation / Variables

Sine rule. Use with non-right triangles when a side and its opposite angle are known.

Cosine rule — finding a side or an angle.

Area of triangle using two sides and included angle.

3.4 — Radians: Arc & Sector

Formula

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Explanation / Variables

Arc length. θ must be in **radians**.

Area of sector. θ in radians.

3.5 & 3.6 — Trigonometric Identities

Formula

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

Explanation / Variables

Definition of tangent in terms of sine and cosine.

Pythagorean identity. Fundamental relation between sin and cos.

Double angle formula for sine.

Double angle formula for cosine — three equivalent forms.

Topic 4: Statistics and Probability

4.2 — Spread

Formula

$$\text{IQR} = Q_3 - Q_1$$

Explanation / Variables

Interquartile range. Spread of the middle 50% of data.

4.3 — Mean

Formula

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n} \quad \text{where } n = \sum_{i=1}^k f_i$$

Explanation / Variables

Mean of a frequency distribution. x_i = value, f_i = frequency.

4.5 — Probability

Formula

$$P(A) = \frac{n(A)}{n(U)}$$

$$P(A) + P(A') = 1$$

Explanation / Variables

Classical probability. Equally likely outcomes.

Complementary events. A' = "not A".

4.6 — Combined Events

Formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Explanation / Variables

Addition rule. Avoid double-counting the overlap.

Mutually exclusive events: $P(A \cap B) = 0$.

Conditional probability: A given B has occurred.

Independent events: occurrence of one does not affect the other.

4.7 — Expected Value

Formula

$$E(X) = \sum_{i=1}^k x_i \cdot P(X = x_i)$$

Explanation / Variables

Weighted average of all outcomes. Mean of a discrete distribution.

4.8 — Binomial Distribution

Formula

$$X \sim B(n, p)$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Explanation / Variables

n = trials, p = probability of success. Requires fixed n , independent trials, constant p .

Mean of a binomial distribution.

Variance. $(1 - p)$ = probability of failure.

4.12 — Normal Distribution**Formula**

$$z = \frac{x - \mu}{\sigma}$$

Explanation / Variables

Standardised normal variable (z -score). μ = mean, σ = standard deviation. Converts to $N(0, 1)$.

Topic 5: Calculus

5.3 — Differentiation: Standard Derivatives

Formula

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

$$f(x) = \sin x \Rightarrow f'(x) = \cos x$$

$$f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

$$f(x) = e^x \Rightarrow f'(x) = e^x$$

$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$$

Explanation / Variables

Power rule. Valid for all real n .

Derivative of sine.

Derivative of cosine. Note the negative sign.

Derivative of e^x is itself.

Derivative of natural logarithm.

5.6 — Differentiation Rules

Formula

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

Quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Explanation / Variables

For composite functions $y = g(u)$, $u = f(x)$.
"Differentiate outside, times derivative of inside."

For a product of two functions. Both terms must be included.

For a quotient. Remember: "low d-high minus high d-low, over low squared."

5.5 & 5.10 — Integration: Standard Integrals

Formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int e^x dx = e^x + C$$

$$A = \int_a^b y dx$$

Explanation / Variables

Reverse power rule. Increase power by 1, divide by new power.

Special case: $n = -1$. Result is natural log.

Integral of sine. Note the negative sign.

Integral of cosine.

Integral of e^x is itself.

Area between $y = f(x)$ and the x -axis, where $f(x) > 0$.

5.9 — Kinematics

Formula

$$v(t) = s'(t) \quad ; \quad a(t) = v'(t) = s''(t)$$

$$s(t) = \int v(t) dt$$

$$\text{Distance} = \int_{t_1}^{t_2} |v(t)| dt$$

$$\text{Displacement} = \int_{t_1}^{t_2} v(t) dt$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Explanation / Variables

Velocity = derivative of displacement.
Acceleration = derivative of velocity.

Displacement from integrating velocity. Add initial condition to find C .

Total distance travelled. Use $|v|$ — split integral at zeros where v changes sign.

Net displacement (can be negative). Different from total distance!

Acceleration as second derivative of displacement.

Note on Calculator Use

In IB AA SL exams, a GDC is permitted in Paper 2 only. Paper 1 is non-calculator.
Show all working clearly — answers without method may not receive full marks.